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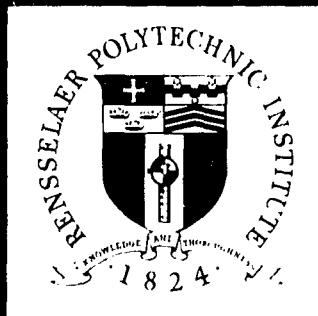
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SUPERSONIC DRAG OF SLENDER BODIES  
in a  
SIMPLE DIATOMIC DISSOCIATING GAS

by

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Air Force Office of Scientific Research  
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## INTRODUCTION

Recently, much has been done on the flow problems where chemical reactions take place in the gaseous mixture.<sup>1-9</sup> In these types of flows, one must consider the chemical reactions as well as the fluid dynamics simultaneously. Clarke<sup>5</sup> has obtained the linearized solution for a flow past a corner in a reacting gas mixture, while Vincenti<sup>2</sup> solved the problem of nonequilibrium flow over a wavy wall and showed that the extra drag due to nonequilibrium effects is positive. Li and Wang<sup>9</sup> have presented a linearized slender-body theory of dissociating gas flows. But no numerical results had been worked out. This paper will be concerned with some quantitative results of linear slender-body theory. In particular, the drag of two cases will be considered; namely, a circular cone and a parabolic spindle. These results should be useful for the development of theories with higher approximations. In fact the extra drag due to nonequilibrium effects turns out to be quite appreciable in the first case. For the case of the circular cone, the extra drag is positive, while for the case of the parabolic spindle, the extra drag is negative indicating that the nonequilibrium effects decrease the total drag.

# GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

In the following we shall be concerned with flows where the only chemical reactions taking place are molecular dissociations and atomic recombinations. By ruling out ionization, we are actually restricting ourselves to a certain temperature range.<sup>(10)</sup> Furthermore, we assume that viscosity, diffusion and heat conduction of the gases are negligible. From this, the continuity equation for the mixture is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0 \quad (1)$$

where  $\rho$  is the density of the mixture and  $\bar{u}$  is the macroscopic velocity vector of the mixture.

The dynamical equation of motion is

$$\frac{D\bar{u}}{Dt} = -\frac{1}{\rho} \nabla P \quad (2)$$

Here  $\frac{D}{Dt}$  is the substantial derivative and  $P$  is the pressure.

The energy equation for the mixture becomes

$$\rho \frac{Dh}{Dt} = \frac{DP}{Dt} \quad (3)$$

where  $h$  is the specific enthalpy of the mixture. From the dynamical equation of motion and the expression

$$H = \frac{1}{2} \bar{u} \cdot \bar{u} + h \quad (4)$$



where  $H$  is defined as the stagnation enthalpy, the above energy equation can also be written as

$$\rho \frac{DH}{Dt} = \frac{\partial P}{\partial t} \quad (5)$$

From the thermodynamic point of view, we assume the different gases in the mixture to be thermally perfect. Thus we can write the thermal equation of state for a dissociating gas as

$$P = (1 + \alpha) \rho \frac{R}{W_2} T \quad (6)$$

where  $\frac{R}{W_2}$  is the molecular gas constant,  $T$  is the temperature of the mixture and  $\alpha$  is the degree of dissociation defined as the mass fraction of gas dissociated. The caloric equation of state is

$$e = e(\alpha, T) \quad (7)$$

where  $e$ , the specific internal energy of the mixture, consists of the specific internal energies of the participating species. For dissociating diatomic gases,

$$e = (1 - \alpha)e_2 + 2\alpha e_1 \quad (8)$$

where  $e_2$  and  $e_1$  are the specific internal energy of the molecular and atomic species respectively. With the assumption that the translational, rotational and vibrational degrees of freedom are fully excited, and the contributions of energy from the other

degrees of freedom are negligible, we can express the specific internal energies derived from statistical thermodynamical considerations as,

$$e_{it} = \frac{3}{2} \frac{R}{W_i} T + \bar{\lambda}_i \alpha d = \frac{3}{2} R_i T + \bar{\lambda}_i \alpha d \quad (9)$$

$$e_{in} = R_i T \quad (10)$$

$$e_{iv} = \frac{\theta}{e^{\theta} - 1} R_i T = f(\theta) R_i T \quad (11)$$

where

$d$  = dissociation energy per unit mass of diatomic gas

$R$  = universal gas constant

$R_i$  = gas constant per unit mass of  $i$ -species

$W_i$  = molecular weight of  $i$ -species

$$\theta = \frac{h^* \nu}{k T}$$

$$f(\theta) = \frac{\theta}{e^{\theta} - 1}$$

$h^*$  = Planck's constant

$k$  = Boltzman's gas constant

$\nu$  = vibrational frequency

$$\bar{\lambda}_i = 1 \text{ when } i = 1$$

$$\bar{\lambda}_i = 0 \text{ when } i = 2$$

and the subscript  $i$  may stand for 1 or 2 for the case of a dissociating diatomic gas. In this case, the caloric equation of state is

$$e = \left[ \frac{5}{2} + f(\theta)(1-\alpha) + \frac{\alpha}{2} \right] \frac{R}{W_2} T + \alpha d \quad (12)$$

The specific enthalpy is expressed by

$$h = \left[ \frac{7}{2} + f(\theta)(1-\alpha) + \frac{3\alpha}{2} \right] \frac{R}{W_2} T + \alpha d \quad (13)$$

In the case of ideal dissociating gas,  $f(\theta) = \frac{1}{2}$ .

The continuity equation for the i-species can be written as

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \bar{u}_i) = \sigma_i \quad (14)$$

where  $\sigma_i$  is the rate of mass product of i-species per unit volume, or usually known as the chemical source term of i-species and  $\bar{u}_i$  is the macroscopic velocity of the i-species. For example, in a dissociating gas, the conservation of atomic gas can be expressed as

$$\frac{\partial(\alpha \rho)}{\partial t} + \nabla \cdot [\rho \alpha (\bar{u} + \bar{V}_A)] = \dot{A} \quad (15)$$

where  $\dot{A}$  is the atomic mass rate production per unit volume, and  $\bar{V}_A$  is the diffusion velocity vector of the atoms. In regions outside the boundary layer or where the flow is inviscid, the term involving  $\bar{V}_A$  can be neglected. Hence

$$\frac{D\alpha}{Dt} = \frac{\dot{A}}{\rho} \quad (15a)$$

From consideration of chemical kinetics, the term on the right hand side of the equation can be obtained. Following J.F. Clarke<sup>4</sup>, we write, for the case of a diatomic, dissociating gas

$$\frac{D\alpha}{Dt} = \frac{4K_{\infty}\rho^2(1+\alpha)}{W_2} \left\{ \frac{K_f W_2}{K_{\infty} 4\rho} (1-\alpha) - \alpha^2 \right\} = \frac{1}{\tau} \left\{ K(1-\alpha) - \alpha^2 \right\} \quad (16)$$

where  $K_f$  and  $K_{\infty}$  are the dissociation and recombination rate constants respectively. Also,

$$\tau = \frac{W_2^2}{4K_{\infty}\rho^2(1+\alpha)} \quad (17)$$

known as the characteristic reaction time, and

$$K = \frac{K_f}{K_{\infty}} \frac{W_2}{4\rho} \quad (18)$$

which can be related to the atom mass fraction under equilibrium conditions in some circumstances.

If we restrict ourselves to steady flow, the above governing equations for inviscid, dissociating, diatomic gas with the stated assumptions become

$$\nabla \cdot (\rho \bar{u}) = 0 \quad (19)$$

$$(\bar{u} \cdot \nabla) \bar{u} = -\frac{1}{\rho} \nabla P \quad (20)$$

$$\bar{u} \cdot \nabla H = 0 \quad (21)$$

$$P = (1+\alpha) \rho \frac{R}{W_2} T \quad (22)$$

$$h = h(\tau, \alpha) \quad (23)$$

$$\bar{u} \cdot \nabla \alpha = \frac{1}{\tau} \left\{ K(1-\alpha) - \alpha^2 \right\} \quad (24)$$

The specific entropy,  $s$ , can be introduced by the equation

$$Tds = dh - \frac{1}{\rho} dP - (\mu_1 - \mu_2) d\alpha \quad (25)$$

where  $\mu_2 - \mu_1$  is the difference of the chemical potentials of the molecules and atoms.

$\mu_1 - \mu_2$  can be related to  $\alpha$  and  $T$  as 4,5

$$\mu_1 - \mu_2 = \frac{RT}{W_2} \ln \left\{ \left( \frac{\alpha}{\alpha_e} \right)^2 \frac{1 - \alpha_e^2}{1 - \alpha^2} \right\} \quad (26)$$

These equations can be combined to yield<sup>11</sup>

$$\frac{1}{a_f^2} (\vec{u} \cdot \nabla)(\vec{u} \cdot \vec{u}) - \nabla \cdot \vec{u} = \lambda \vec{u} \cdot \nabla \alpha \quad (27)$$

where

$$a_f^2 = \left( \frac{\partial P}{\partial \rho} \right)_{s, \alpha} = - \frac{\left( \frac{\partial h}{\partial \rho} \right)_{P, \alpha}}{\left[ \left( \frac{\partial h}{\partial \rho} \right)_{P, \alpha} - \frac{1}{\rho} \right]} \quad (28)$$

is the frozen speed of sound, and

$$\lambda = - \frac{\left( \frac{\partial h}{\partial \alpha} \right)_{P, \rho}}{\rho \left( \frac{\partial h}{\partial \rho} \right)_{P, \alpha}} = - \frac{1}{\rho} \left( \frac{\partial \rho}{\partial \alpha} \right)_{P, T} - \rho \beta_f \frac{1}{C_{Pf}} \left( \frac{\partial h}{\partial \alpha} \right)_{P, T} \quad (29)$$

In the above expression

$$C_{Pf} = \left( \frac{\partial h}{\partial T} \right)_{P, \alpha} \quad (30)$$

is the specific heat at constant pressure and frozen composition and

$$\beta_f = - \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial T} \right)_{P, \alpha} \quad (31)$$

is the volume expansion coefficient at constant pressure and frozen composition. The problem now is to solve equations (24) and (27) simultaneously, subject to the proper boundary conditions; namely, that the flow be tangent to the body on the surface of the body and that there be no chemical reaction taking place between the gas and the body.

### Linearization

For flow past slender bodies, the disturbances are very small. To simplify the problem of steady flow of a simple dissociating gas mathematically, we may assume the disturbances to a free stream in chemical equilibrium to be small and proceed to linearize the equations. We assume that not only will the perturbation velocities be small, but the deviations of  $\alpha$ ,  $\alpha_f$ ,  $\alpha_e$  from their respective free stream values will also be small. Let  $u'$ ,  $v'$  and  $w'$  be the non-dimensional perturbation velocity potential defined as

$$\begin{aligned} u' &= \frac{u}{u_\infty} - 1 \\ v' &= \frac{v}{u_\infty} \\ w' &= \frac{w}{u_\infty} \end{aligned} \tag{32}$$

In the linearization, we neglect terms involving squares or products of the perturbation quantities. The validity of these assumptions holds for slender bodies. With these approximations,

equations (24) and (27) can be combined to give<sup>11</sup>

$$\chi^{-1} \frac{\partial}{\partial \xi} \left[ (1 - M_{f\infty}^2) \frac{\partial u'}{\partial \xi} + \frac{\partial v'}{\partial \eta} + \frac{\partial w'}{\partial \zeta} \right] + \left[ (1 - M_{e\infty}^2) \frac{\partial u'}{\partial \xi} + \frac{\partial v'}{\partial \eta} + \frac{\partial w'}{\partial \zeta} \right] = 0 \quad (33)$$

where

$$\begin{aligned} M_{f\infty} &= \frac{u_{\infty}}{a_{f\infty}} \\ M_{e\infty} &= \frac{u_{\infty}}{a_{e\infty}} \\ \xi &= \frac{x}{L} \\ \eta &= \frac{y}{L} \\ \zeta &= \frac{z}{L} \end{aligned} \quad (34)$$

and

$$\chi^{-1} = \frac{\tau_{\infty} u_{\infty}}{L} \quad (35)$$

$\chi$  stands for the ratio of characteristic flow time and chemical relaxation time.  $L$  is a suitable characteristic length of the flow. Within the linearized theory,<sup>11</sup> equation (33) can be written as

$$\chi^{-1} \frac{\partial}{\partial \xi} \left[ (1 - M_{f\infty}^2) \frac{\partial \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial^2 \phi}{\partial \zeta^2} \right] + \left[ (1 - M_{e\infty}^2) \frac{\partial \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial^2 \phi}{\partial \zeta^2} \right] = 0 \quad (36)$$

where the non-dimensional velocity potential  $\phi$  has been introduced.

The non-dimensional perturbation velocities are given as

$$u' = \frac{\partial \phi}{\partial \xi}, \quad v' = \frac{\partial \phi}{\partial \eta}, \quad w' = \frac{\partial \phi}{\partial \zeta} \quad (37)$$

The solution of equation (36) subject to the linearized boundary conditions has been solved by Li and Wang<sup>9</sup> by using the method of Laplace and Fourier Transforms.

### Drag

From the expression for  $\phi$ , the perturbation velocities, surface pressure and hence the drag coefficient for slender bodies can be obtained as<sup>9</sup>

$$\begin{aligned} \frac{D}{\frac{1}{2} \rho_\infty U_\infty^2} = & -\frac{1}{2\pi} \int_0^l \int_0^l S''(x) S''(\xi) \log |x-\xi| d\xi dx \\ & + \frac{S'(l)}{2\pi} \int_0^l S''(\xi) \log(l-\xi) d\xi - \frac{1}{U_\infty^2} \left( \oint_C \phi \frac{\partial \phi}{\partial n} ds \right)_{x=l} + C_{p0} S(l) \\ & + \frac{1}{2\pi} \int_0^l \int_0^x S''(x) S'(\xi) \frac{e^{-\beta_1(x-\xi)} - e^{-\beta_2(x-\xi)}}{x-\xi} d\xi dx \\ & - \frac{S'(l)}{4\pi} \int_0^l S'(\xi) \frac{e^{-\beta_1(l-\xi)} - e^{-\beta_2(l-\xi)}}{l-\xi} d\xi \end{aligned} \quad (38)$$

Where  $S(x)$  denotes the cross sectional area of the body and should cause no confusion with the specific entropy. The first four terms are the same as those obtained by Ward and the last two terms are purely due to non-equilibrium effect. The third term can be obtained when specific body shapes are considered. In particular, for bodies of revolution and omitting the base drag for bodies not pointed at both ends, the expression for the drag<sup>9</sup> becomes



$$\begin{aligned}
\frac{D}{\frac{1}{2} \rho_\infty U_\infty^2} = & -\frac{1}{2\pi} \int_0^l \int_0^l S''(x) S''(\xi) \log |x-\xi| d\xi dx \\
& + \frac{S'(l)}{\pi} \int_0^l S''(\xi) \log(l-\xi) d\xi - \frac{[S'(l)]^2}{2\pi} \log \frac{\sqrt{M_{f0}^2 - 1}}{2} R(l) \\
& + \alpha_0^2 S(l) + \frac{1}{2\pi} \int_0^l \int_0^x S''(x) S'(\xi) \frac{e^{-\beta_1(x-\xi)} - e^{-\beta_2(x-\xi)}}{x-\xi} d\xi dx \\
& - \frac{S'(l)}{\pi} \int_0^l S'(\xi) \frac{e^{-\beta_1(l-\xi)} - e^{-\beta_2(l-\xi)}}{l-\xi} d\xi
\end{aligned} \tag{39}$$

where  $\alpha_0$  is the angle of attack, and

$$\begin{aligned}
\beta_2 &= \frac{M_{f0}^2}{\tau_\infty U_\infty M_{e\infty}^2} \\
\beta_1 &= \frac{M_{f0}^2}{\tau_\infty U_\infty M_{e\infty}^2} \left[ \frac{M_{e\infty}^2 - 1}{M_{f0}^2 - 1} \right]
\end{aligned}$$

The last two terms in the above expression (Eq. 39) gives the extra drag due to nonequilibrium effects. The relative importance of the drag will be investigated for two cases; namely, the unyawed circular cone and the parabolic spindle.

### Examples

In the following examples, computations are based on the values  $B^2 = \frac{B_e^2}{B_f^2} = 1.5$  and  $\left( \frac{a_{f0}^2}{a_{e\infty}^2} \right) = 1.35$ . Correspond to these values, the equilibrium and frozen Mach numbers are 2.12 and 1.83 respectively.

Case a) Unyawed cone

The profile of the cone is given by (see figure 1)

$$\tan \delta = \frac{\bar{R}}{x} \quad (40)$$

where  $\delta$  is the semi-vertex angle of the cone. Hence,

$$S(x) = \pi \bar{R}^2 = \pi x^2 \tan^2 \delta \quad (41)$$

Knowing  $S'(x)$  and  $S''(x)$  the drag of a cone in a simple dissociating gas can be calculated from equation (39) by double integration. For this case, denoting  $D^{(1)}$  and  $D^{(2)}$  as the drag of the cone in an inert gas and the extra drag due to chemical reaction, the expression for  $\frac{D^{(2)}}{\frac{1}{2} \rho_\infty u_\infty^2}$  turns out to be

$$\begin{aligned} \frac{D^{(2)}}{\frac{1}{2} \rho_\infty u_\infty^2} = & 2\pi \tan^4 \delta \left\{ \frac{1}{2} [E_i(\beta_2 l) - E_i(\beta_1 l)] \right. \\ & + \frac{1}{\beta_1 l} - \frac{1}{\beta_2 l} + \frac{1 - e^{-\beta_1 l}(\beta_1 l + 1)}{2(\beta_1 l)^2} - \frac{1 - e^{-\beta_2 l}(\beta_2 l + 1)}{2(\beta_2 l)^2} \\ & \left. + \left[ \frac{1}{(\beta_1 l)^2} + \frac{1}{(\beta_1 l)} \right] (1 - e^{-\beta_1 l}) - \left[ \frac{1}{(\beta_2 l)^2} + \frac{1}{(\beta_2 l)} \right] (1 - e^{-\beta_2 l}) \right\} \quad (42) \end{aligned}$$

The expression for  $\frac{D^{(1)}}{\frac{1}{2} \rho_\infty u_\infty^2}$  can be also integrated and the result obtained as

$$\frac{D^{(1)}}{\frac{1}{2} \rho_\infty u_\infty^2} = 2\pi l^2 \tan^4 \delta \left[ -\frac{1}{2} - \log \left( \tan \delta \frac{\sqrt{M_f^2 - 1}}{2} \right) \right] \quad (43)$$

$\beta_2 = \frac{M_{f\infty}^2}{\tau_\infty u_\infty M_{e\infty}^2}$  is essentially the inverse characteristic relaxation distance. For a model with length  $l$  equal to the characteristic relaxation distance, i.e.,  $\beta_2 l = 1$ , the ratio

$D^{(2)}/D^{(1)}$  has been calculated for  $\delta = 5^\circ$  to  $30^\circ$  at  $5^\circ$  intervals. The result is given in table 1. For a model of fixed length, the effect of chemical reaction is seen to decrease rapidly for increasing  $\beta_2$ . (Table 2).

Case b) Parabolic spindle at zero angle of attack (Fig. 2).

The profile of this body is given by the equation

$$\bar{R}(x) = \delta \frac{x}{l} (l-x) = \delta x \left(1 - \frac{x}{l}\right) \quad (44)$$

Hence

$$S(x) = \pi \bar{R}^2 = \pi \delta^2 x^2 \left(1 - \frac{x}{l}\right)^2 \quad (45)$$

From the expression for drag given by equation (39), and noting that  $S'(1) = 0$ ,

$$\begin{aligned} \frac{D^{(2)}}{\frac{1}{2} \rho_\infty u_\infty^2} = & 2\pi \delta^4 l^2 \left\{ 24 \left[ \frac{1}{(\beta_1 l)^6} - \frac{1}{(\beta_2 l)^6} \right] - 3 \left[ \frac{1}{(\beta_1 l)^4} - \frac{1}{(\beta_2 l)^4} \right] \right. \\ & + \frac{1}{2} \left[ \frac{1}{(\beta_1 l)^2} - \frac{1}{(\beta_2 l)^2} \right] - \frac{1}{5} \left[ \frac{1}{(\beta_1 l)} - \frac{1}{(\beta_2 l)} \right] \\ & + e^{-\beta_1 l} \left[ -\frac{1}{(\beta_1 l)^3} - \frac{9}{(\beta_1 l)^4} - \frac{24}{(\beta_1 l)^5} - \frac{24}{(\beta_1 l)^6} \right] \\ & \left. - e^{-\beta_2 l} \left[ -\frac{1}{(\beta_2 l)^3} - \frac{9}{(\beta_2 l)^4} - \frac{24}{(\beta_2 l)^5} - \frac{24}{(\beta_2 l)^6} \right] \right\} \quad (46) \end{aligned}$$

and

$$\frac{D^{(1)}}{\frac{1}{2} \rho_{\infty} u_{\infty}^2} = 4\pi \delta^4 \ell^2 \left( \frac{1}{24} \right) \quad (47)$$

The ratio of  $D^{(2)}/D^{(1)}$  for  $\beta_2 \ell = 1$  is shown in table 3. In this case this ratio is independent of the slenderness ratio.

### Conclusion

For the case of the cone,  $D^{(2)}$  turns out to be positive and is quite appreciable compared to  $D^{(1)}$ . This seems to be in support with the reasoning in Kusakawa and Li's report;<sup>12</sup> namely, the presence of chemical reaction will increase the entropy just like in the case of vorticity. The ratio  $D^{(2)}/D^{(1)}$  for  $\beta_2 \ell = 1$  increases quite rapidly for increasing semi-vertex angle of the cone. In particular, the value already reaches 10% for a cone of semi-vertex angle of between  $15^\circ$  and  $20^\circ$ . This result justifies the investigation of theories of non-equilibrium flow with higher approximations. These higher order theories are useful not only in the increase of accuracy, but also in their applicability to a wider range of problems. For example, by the higher order theories, we are able to extend the Mach number range so as to approach the hypersonic case. The effects of chemical reactions decreases very rapidly for a given model if  $\beta_2$  is increased as shown in table 2.

In the case of a parabolic spindle,  $D^{(2)}$  is negative and is small compared to  $D^{(1)}$ . From thermodynamic considerations, it has been shown in Ref. 12 that the extra drag caused by chemical equilibrium will be positive for a subsonic flow past a body. In order to relate the subsonic and the supersonic cases for slender pointed bodies, the following expression for the extra drag obtained from reference 9 can be used

$$\frac{D^{(2)}}{\frac{1}{2} \rho_{\infty} u_{\infty}^2} = \frac{1}{2\pi} \int_0^l \int_0^x S''(x) S'(\xi) \left[ \frac{e^{-\beta_1(x-\xi)} - e^{-\beta_2(x-\xi)}}{x-\xi} \right] d\xi dx \quad (48)$$

for both supersonic and subsonic case. Since  $\beta_1 < \beta_2$  for subsonic case and  $\beta_1 > \beta_2$  for supersonic case, the sign within the bracket is reversed for the two cases. Hence, for the same body, if the subsonic drag due to chemical reactions is positive, its counterpart in the supersonic case must be negative. It was shown<sup>11</sup> that in a medium where chemical reaction is taking place, disturbances are being damped. Hence, in a supersonic flow, the wave drag in flow of a reacting gas past a body will be less than that of a non-reacting gas. As pointed out at the end of reference 12, this phenomena would have to be included in the discussion of the extra drag from effects of chemical reaction. The results on the parabolic spindle indicates that the decrease of wave drag is sufficiently large to cause  $D^{(2)}$  to be negative. Unlike in the case of the cone, where only dissociation takes place, in the case of parabolic spindle both dissociation and recombination

occurs. In addition to this difference, the ratio  $D^{(2)}/D^{(1)}$  in the latter case is independent of the slenderness ratio. For a fixed  $\ell$ , as in the case of the cone, the effects due to chemical reaction decreases rapidly for increasing  $\beta_2$ , which may be interpreted as the inverse characteristic relaxation distance. The calculations in this paper are based on  $M_{f,\infty} = 1.83$  and  $M_{e,\infty} = 2.12$  so that the linearized theory is applicable. The results as indicated here may be quite different from the hypersonic slender body behavior. For that purpose, higher order theories with a higher Mach number range of applicability must await future development.

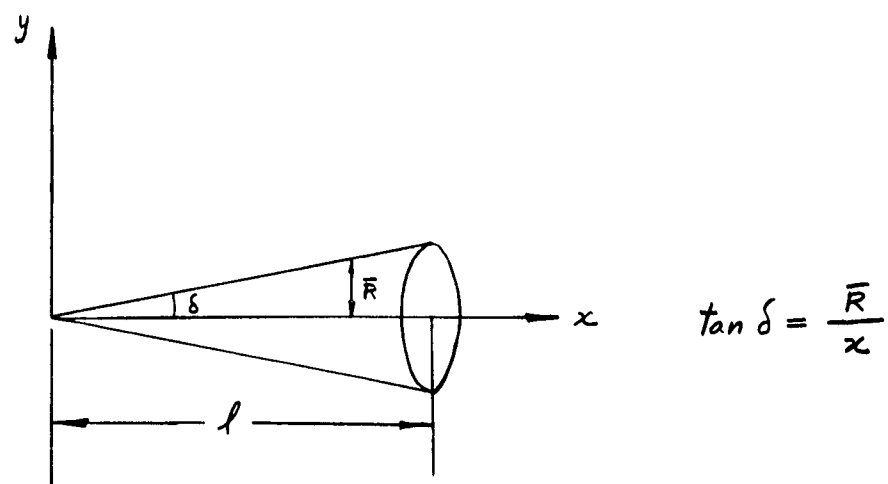


Figure 1: Unyawed Cone

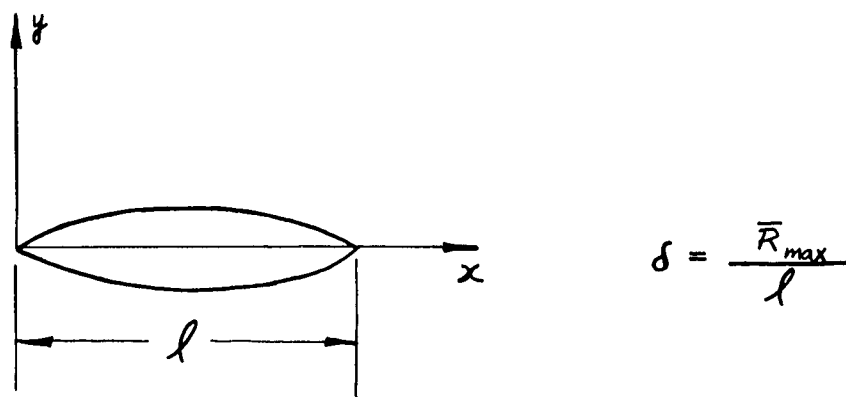


Figure 2: Parabolic Spindle

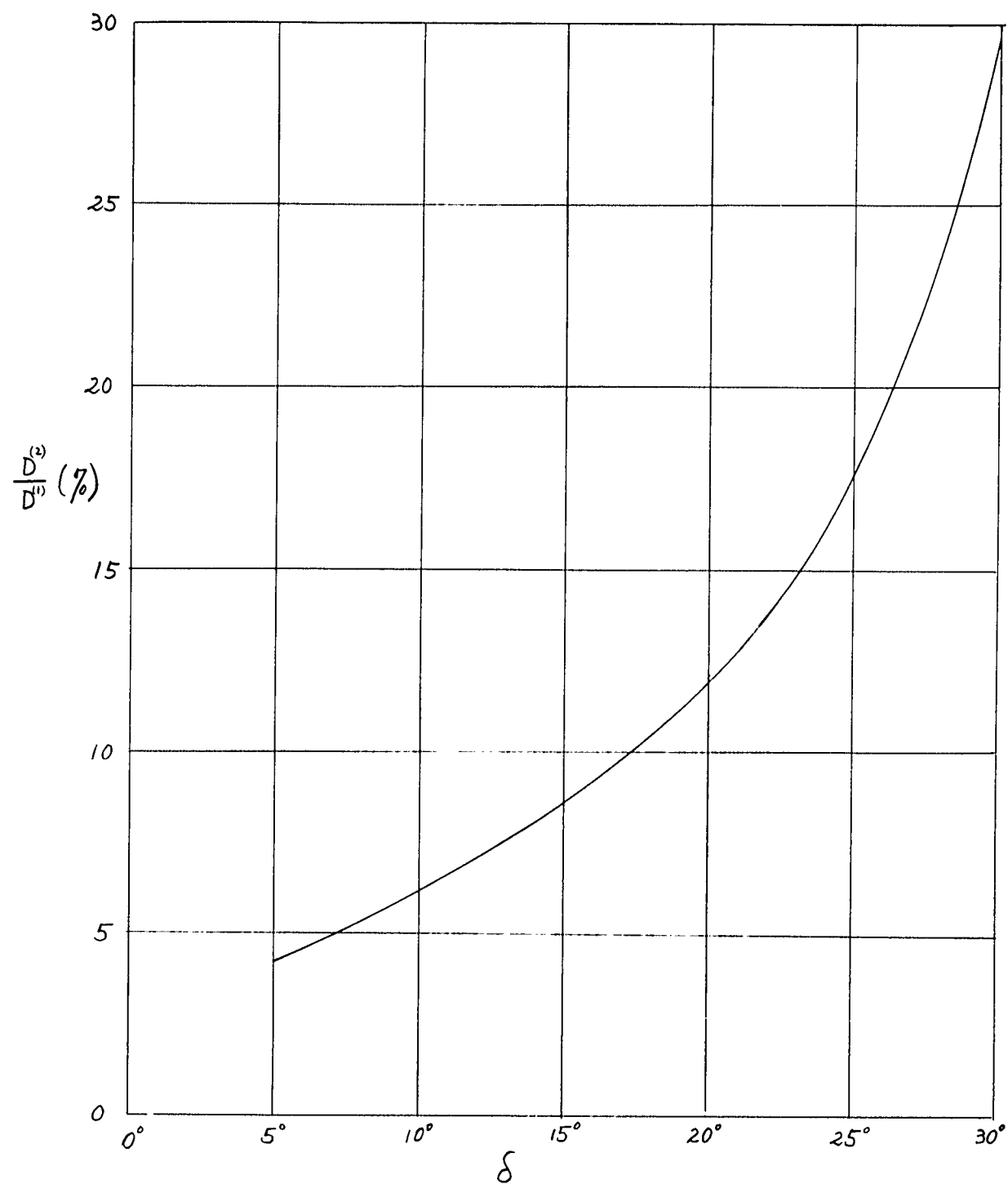


Fig. 3: Variation of  $\frac{D^{(2)}}{D^{(1)}}$  for an unyawed cone with  $\delta$  .



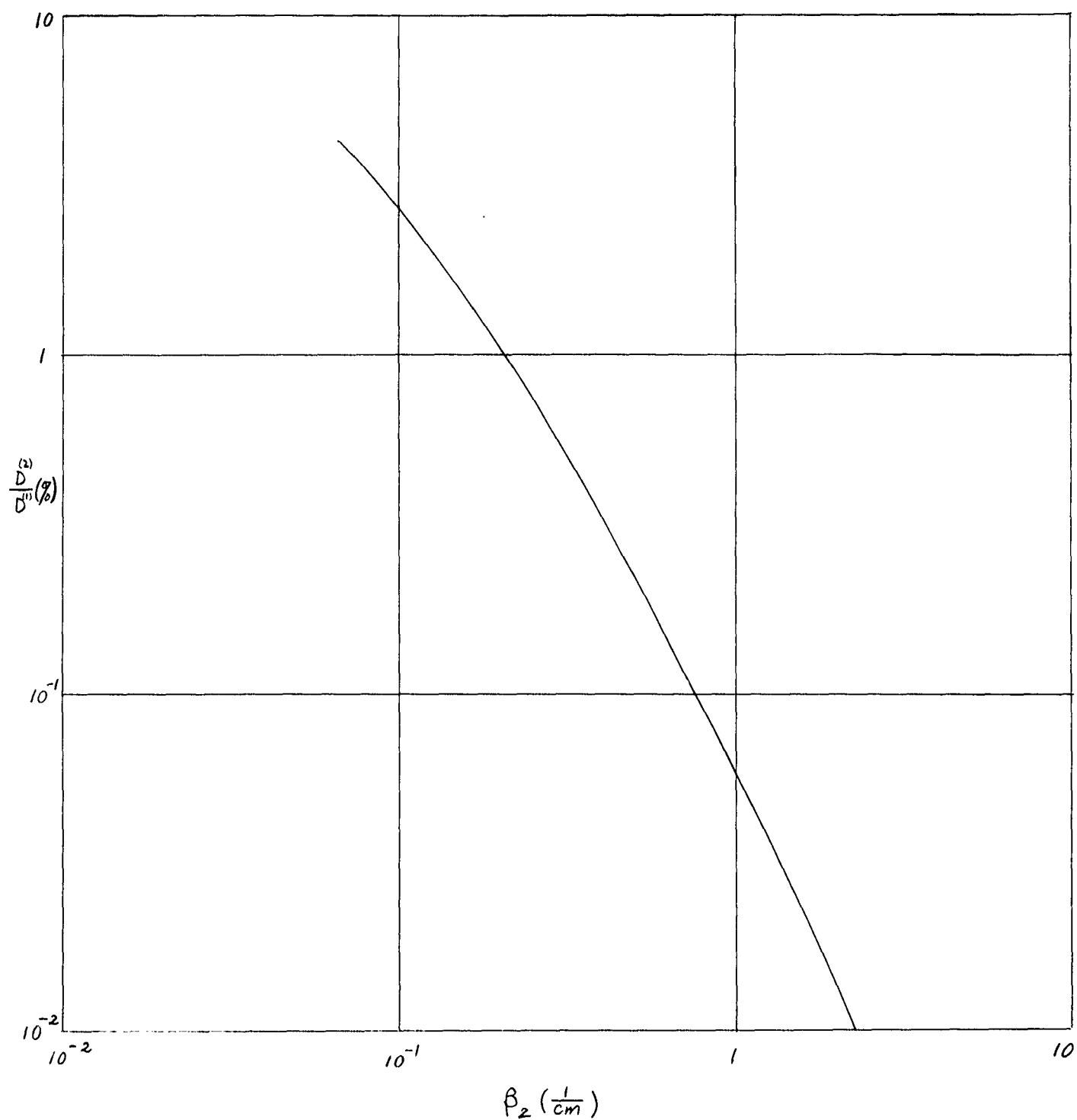


Fig. 4: Variation of  $\frac{D^{(2)}}{D^{(1)}}$  for an unyawed cone of  $\ell = 15 cm$  with  $\beta_2$ .  
 $\delta = 5^\circ$

Table 1: Variation of  $D^{(2)}/D^{(1)}$  with  $\delta$  for an unyawed cone.

$$\beta_2 l = 1$$

$$l = 15 \text{ cm}$$

$\delta$	$5^\circ$	$10^\circ$	$15^\circ$	$20^\circ$	$25^\circ$	$30^\circ$
$\frac{D^{(2)}}{D^{(1)}} \text{ in } \%$	4.3	6.2	8.6	12.1	17.7	29.7

Table 2: Variation of  $D^{(2)}/D^{(1)}$  with  $\beta_2$  for an unyawed cone.

$$l = 15 \text{ cm}$$

$$\delta = 5^\circ$$

$\beta_2$	$\frac{1}{15}$	.0715	.715	7.15
$\frac{D^{(2)}}{D^{(1)}} \text{ in } \%$	4.3	4.0	.11	.11 x $10^{-2}$

Ratio of  $D^{(2)}/D^{(1)}$ 

for a

Table 3 parabolic spindle.

$$\beta_2 l = 1$$

$$\left| \frac{D^{(2)}}{D^{(1)}} \right| = \frac{.000109}{.081} = .14\%$$

## SYMBOLS USED

$a_f$	frozen speed of sound
$a_e$	equilibrium speed of sound
$\dot{A}$	atomic mass rate production per unit volume
$C_{Pf}$	specific heat at constant pressure and frozen composition
$D$	drag
$D''$	drag of flow past a body in inert gas
$D^{(2)}$	extra drag due to chemical reaction
$d$	dissociation energy per unit mass of diatomic gas
$e$	specific internal energy of the mixture
$e_2$	specific internal energy of the molecular species
$e_1$	specific internal energy of the atomic species
$h$	specific enthalpy of the mixture
$h^*$	Planck's constant
$H$	stagnation enthalpy
$k$	Boltzman's constant
$K_f$	dissociation rate constant
$K_r$	recombination rate constant
$L$	characteristic length in the flow
$M_{f_\infty}$	frozen free-stream Mach number
$M_{e_\infty}$	equilibrium free-stream Mach number
$P$	pressure of the mixture
$R$	universal gas constant
$R_i$	gas constant per unit mass of $i$ -species

$\bar{R}$  radial distance on the body profiles

$S$  specific entropy

$S(x)$  cross sectional area of the body

$t$  time

$T$  temperature of the mixture

$\bar{u}$  macroscopic velocity vector of the mixture

$\bar{u}_i$  macroscopic velocity vector of  $i$ -species

$\bar{V}_A$  diffusion velocity vector of the atoms

$u', v', w'$  non-dimensional perturbation velocities

$W_2$  molecular weight of undissociated gas

$W_i$  molecular weight of the  $i$ -species

$\alpha_0$  angle of attack

$\alpha$  degree of dissociation

$\alpha_e$  degree of dissociation at equilibrium flow

$\beta_f$  volume expansion coefficient at constant pressure  
and frozen composition

$\delta$  semi-vertex angle for the circular cone<sup>or</sup><sub>A</sub> slenderness  
ratio of the parabolic spindle

$\chi$  ratio of characteristic flow time and chemical  
relaxation time

$\tau$  characteristic reaction time

$$\theta = \frac{h^* \nu}{kT}$$

$\rho$  density of the mixture

$\nu$  vibrational frequency

$\lambda$  (see equation 29)

$$\bar{\lambda}_i = \begin{cases} 1 & \text{when } i = 1 \\ 2 & \text{when } i = 2 \end{cases}$$

$\mu_1$  chemical potential of atoms

$\mu_2$  chemical potential of molecules

$$B^2 = \frac{B_e^2}{B_f^2}, \quad B_e = \sqrt{M_{e\infty}^2 - 1}, \quad B_f = \sqrt{M_{f\infty}^2 - 1}$$

$$\beta_1 = \frac{M_{f\infty}^2}{T_\infty u_\infty M_{e\infty}^2} \left[ \frac{M_{e\infty}^2 - 1}{M_{f\infty}^2 - 1} \right], \quad \beta_2 = \frac{M_{f\infty}^2}{T_\infty u_\infty M_{e\infty}^2}$$

$$f(\theta) = \theta / (e^\theta - 1)$$

$$K = \frac{K_f}{K_v} \frac{W_2}{4\rho}$$

$$\xi = x/L, \quad \eta = y/L, \quad \zeta = z/L$$

#### SUBSCRIPTS

$t$  translational

$r$  rotational

$v$  vibrational

$f$  frozen

$e$  equilibrium

$\infty$  free-stream

$2$  molecular species

$1$  atomic species

$i$  i-species

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